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O.R. Applications

## Reliability and heterogeneity of railway services

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### Abstract

Reliability is one of the key factors in transportation, both for passengers and for cargo. This paper examines reliability in public railway systems. Reliability of railway services is a complex matter, since there are many causes for disruptions and at least as many causes for delays to spread around in space and time.

One way to increase the reliability is to reduce the propagation of delays due to the interdependencies between trains. In this paper we attempt to decrease these interdependencies by reducing the running time differences per track section and by thus creating more homogeneous timetables. We also introduce two heuristic measures, that can be used to evaluate the homogeneity of a timetable.

Because of the complexity of railway systems, we use network wide simulation for the analysis of the alternative timetables. We report on both theoretical and practical cases. Besides a comparison of different timetables, also general timetabling principles are deduced.

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### 1. Introduction

Railway infrastructure capacity is limited and has to be used carefully therefore. Due to the in-

creased utilization of the railway infrastructure in the Netherlands over the past years, the railway system has become quite vulnerable to disruptions. This resulted in a lower punctuality and in many customer complaints.

In the Netherlands, the railway system is characterized by many interconnected services. Passenger transfers, rolling stock circulations and crew schedules all play their role in the relations between trains. However, the shared use of the same

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infrastructure by different railway services, with different origins and destinations, different speeds, and different halting patterns, is probably the main reason for the propagation of delays throughout the network. This makes such a national railway system much more vulnerable to disruptions than metro systems.

Reliability is, together with door-to-door travel time, one of the predominant performance measures in railway traffic. The predictability of the arrival times is a big factor in deciding to use rail or road transport, both for passengers and for cargo.

In this paper we investigate the effect of heterogeneity of the timetable on the reliability of the railway system. This is a subject on which a lot of practical intuition exists, but hardly any scientific literature can be found. Railway traffic is considered to be *heterogeneous* when trains have large running time differences on the same track sections. When running times per track section are more or less equal for all trains, then the timetable is called *homogeneous*. The research goal of this paper implies that we are mainly interested in relatively small initial (primary) disturbances, because no reasonable timetable is robust enough to handle large disturbances or disruptions without severe on-line adjustments of the railway traffic.

Many characteristics and details of a railway timetable have their influence on reliability. Therefore, to understand the relations properly, one first needs to know how a timetable is constructed. Starting from the market demand, line planning is the first phase in railway planning, where train connections are determined: starting and ending stations of lines are chosen, including the routes, and the stations for alighting and boarding. The chosen lines determine the service differentiation—intercity trains, local trains, cargo, etc.—and consequently a large part of the heterogeneity of the timetable. The line planning step is followed by the timetabling step, where departure and arrival

times are chosen. There are often several iterations between these two steps, because a preferred line plan does not imply a feasible timetable. Feedback loops may have to be executed (see Fig. 1).

When the timetable is finished, the rolling stock circulation is planned. This step also includes shunting and scheduling repositioning trips. Both for the regular trains and for the shunting work, train drivers have to be scheduled. For passenger trains, also conductors are needed. Although the steps presented in the flow-diagram in Fig. 1 depend on each other in the presented order, coordination between these steps is not easy. This means that not all forward consequences of a certain planning step are taken into account immediately. For example, in the timetabling phase, one cannot always foresee the resulting impact on the rolling stock circulations nor on the crew schedules. Again, feedback loops may be necessary.

The line plan and the timetable determine to a large extent the heterogeneity of the railway system. Since we investigate the influence of the heterogeneity on reliability and punctuality, we focus on the line planning and the timetabling steps.

In this paper we develop timetables which mainly differ from each other in heterogeneity. While keeping the numbers of stops per hour as much as possible equal for all stations, we develop more heterogeneous and more homogeneous timetables both for a theoretical case and for a practical case.

To quantify the differences between these timetables, two new heterogeneity measures are proposed. These new measures follow the ideas described by Carey (1999), but they also take into account several other characteristics of the timetable. These two heterogeneity measures are not only used for the evaluation of the timetables, but also for the prediction of the reliability.

In this paper, detailed simulation of the railway systems is used for the comparison of the heterogeneous and the homogeneous timetables. A wide



Fig. 1. Sequence of interdependent railway planning phases.

range of disturbance distributions and disturbance levels is used for this evaluation.

Because cargo trains cover less than 10% of the Dutch railway traffic, we concentrate on passenger trains in this paper. Furthermore, we assume, as is common in several European countries, that the timetables are cyclic. This means that passenger services are repeated every cycle time, typically every hour.

When researching heterogeneity, double track sections are more relevant than single track sections or sections with four parallel tracks. Indeed, the timetable for single-track lines is mostly dictated by the distances between passing points. In the case of four parallel tracks, trains with different speeds are usually already separated, and each track has its own speed: one track for slow traffic, one for fast traffic. The interesting part is where all trains for one direction run on one track: double track lines. Notice that many of Europe's main lines are double track lines indeed.

This paper starts with an introduction on railway reliability, including a literature overview. This is followed in Section 3 by a discussion on heterogeneity and its influence on reliability. Also the new heterogeneity measures are introduced here. Sections 4 and 5 present a theoretical and a practical case, respectively. The principle of homogenization is further discussed in Section 6. Conclusions follow in Section 7.

## 2. Punctuality and reliability

When investigating railway reliability, it is important to make a distinction between *primary* and *secondary* delays. Primary delays are initial delays caused on a train from the outside and *not* by other trains. These delays are caused by malfunctioning rolling stock, malfunctioning infrastructure, bad weather conditions, excessive alighting and boarding times of passengers, accidents at road-railroad crossings, and so on. Secondary delays are those delays of trains that are caused by earlier delays of other trains. They are also referred to as *knock-on delays*. Secondary delays appear because of the shared use of the same infrastructure, rolling stock connections,

transfers in crew schedules, passenger transfers, dispatching actions, and so on. In our study we consider the primary delays as given, and we aim at developing timetables which both absorb primary delays fast and cause as few secondary delays as possible.

The measures which are chosen for evaluating the reliability are the average delays and the observed punctuality. Punctuality is probably the most widely used reliability measure in practice (Schaafsma, 2001), both in the Netherlands and abroad. This measure calculates the percentage of trains arriving within a certain number of minutes from the scheduled arrival time. In practice in the Netherlands, a 3-minute margin is used. However, in most other countries—as well as for international comparisons—a 5 minute margin is more common.

Besides the arrival punctuality, also the departure or the start-up punctuality can be calculated. Furthermore, it is important at which stations the punctuality is measured.

In simulation research, it is quite easy to compare punctualities on different punctuality margins and on different sets of stations or train types or lines. Other possible measures for reliability, which are not considered in this paper, are the percentage of realized passenger transfers and the average delays of the passengers.

### 2.1. Literature overview

Over the last 15 years, a wide range of researchers has studied railway timetabling and punctuality issues. Literature reports on different types of timetable evaluation models. In the following, we describe first the analytical delay models, starting with max-plus algebra. Thereafter, stochastic models are described, and finally we focus on railway simulation.

Max-plus algebra is an analytical approach for evaluating the robustness of a timetable. Some relevant key characteristics, such as the minimum cycle time, can be calculated with max-plus algebra (Subiono, 2000; Goverde and Soto y Koelemeijer, 2000; Van Egmond, 2000; De Kort, 2000). PETER, an analysis tool based on max-plus algebra, is a performance evaluator for timetables (Soto y

Koelemeijer et al., 2000; Goverde and Odijk, 2002). Whereas max-plus algebra cannot handle stochastic elements, Hansen (2000) uses both queuing theory and max-plus algebra to study the capacity and stability of railway systems, but only in stations.

Huisman, Boucherie and Van Dijk developed a stochastic analytical waiting line model for analyzing delays at a double track section (Huisman and Boucherie, 2001; Huisman et al., 2002). Their models are based on train frequencies and running times only, not on detailed timetables with arrival and departure times.

Higgins, Kozan and Ferreira come up with a model to quantify the risk of delays on a single track line (Higgins et al., 1995). Higgins and Kozan also developed an analytical model to quantify the expected delays of individual passenger trains in an urban rail network (Higgins and Kozan, 1998).

Carey and Kwiecieński (1995) mainly focus on running time supplements (i.e. recovery time) in their stochastic analysis. Carey also uses heuristic measures for timetable reliability (Carey, 1999) and includes behavioral response (Carey, 1998). These approaches are rather simplified, and they lack verification with reality.

Other researchers use simulation as a tool to analyze the influence of delays on the train circulation, given some traffic scenario. SIMON is a Swedish software tool using simulation of the whole network (Wahlborg, 1996; Bergmark, 1996). Amongst others, VirtuOS (König, 2001) and SABINE (Fischer et al., 1995) are used in Germany, and Open Track (Hürlimann, 2001) is a railway simulation program developed at ETH Zürich. UX-SIMU is used for simulation of railway traffic in Denmark (Kaas, 2000).

This literature is mainly focused on the simulation software itself and sometimes on a simple comparison of multiple timetables. More thorough research of the impact of timetabling principles on reliability is scarce.

However, Carey and Carville (2000) focus on the evaluation of delay propagation in large stations by simulation, and on the improvement of the underlying timetables (Carey and Carville, 2003).

Moreover, Middelkoop and Bouwman describe the use of SIMONE (Middelkoop and Bouwman, 2000, 2001) for the evaluation of traffic scenarios in the Netherlands. SIMONE is capable of simulating the entire Dutch railway network. In this paper we also use SIMONE to execute theoretical analyses on the basis of simulation.

### 3. Homogeneity, heterogeneity, and headways

Railway traffic is considered to be homogeneous if all trains have similar characteristics; especially the *same average speed* per track segment, resulting from the running times and the stopping times. Appropriate examples of homogeneous railway traffic are metro systems where all trains have the same running times per track and where all trains stop at all stations. However, for national railway networks, railway traffic cannot be fully homogeneous. Usually cargo trains and passenger trains share the same infrastructure. But probably a more important factor is a large differentiation in passenger services, ranging from short distance trains—which dwell at nearly all stations under-way—via intercity trains, to international high speed connections—with high speeds, only stopping at a few large stations—partly sharing the same infrastructure. If there are large differences in the timetable characteristics of the trains on the same track, then the railway traffic is called heterogeneous.

#### 3.1. Possibilities for homogenization

Homogenization of a railway system means that differences in running times per track section of different trains along a railway line are decreased. There are several alternative options for homogenization.

- Slowing down long distance trains, by adding extra running time supplement.
- Speeding up short distance services, by decreasing running time supplements or by using faster rolling stock.
- Overtaking: in case of overtaking, running time differences should only be regarded between

two consecutive overtaking stations. Only these time differences use additional infrastructure capacity.

- Shorter lines for the short distance services: the running time differences should only be regarded over the reduced length of the line.
- Equalizing the number of stops per train: shift some stops from the short distance services to the long distance services.

### 3.2. Heterogeneity and reliability

When heterogeneous services share the same infrastructure over large distances, timetabling becomes very complicated. Heterogeneity usually leads to many small headway times, which may increase delay propagation in the operations. It is generally accepted in practice that the heterogeneity resulting from the line plan and the timetable has a negative influence on the punctuality and the reliability of a railway system (UIC, 2004).

The current paper provides a scientific analysis of this problem area. To that end, we first develop two measures to quantify the heterogeneity and to predict the resulting reliability. Then simulation of both theoretical and practical cases is used to show the importance of homogeneity of a timetable. Besides the fact that we use the heterogeneity measures in this paper for a theoretical comparison of different timetables, it is also intended to be useful for the development of timetables for real world operations.

### 3.3. Heterogeneity measures

Given the train frequency of a line, the average headway at a location along that line is simply equal to the cycle time divided by the frequency. More useful headway measures are described by Carey (1999). He shows that equalizing scheduled headways for one station has a positive influence on the punctuality when train delay distributions are equal for all trains and sloping downward. The measures he describes are based on this principle. These measures include:

- the percentage of headways smaller than a certain size;

- the percentiles of the headway distribution;
- range, standard deviation, variance, or mean absolute deviation of the headways.

The further description of these measures in the mentioned paper implies that the headways are measured at one single location only.

An important disadvantage of measuring headways at only one single location is that this does not tell anything about the behavior of the trains on the surrounding track sections. Therefore we consider the smallest headways between two consecutive trains on a certain track section instead of at one single location. When the trains on a certain track section are completely homogeneous, then the smallest headway between two consecutive trains is equal to the headway anywhere on this track section. The sum of these minimal headways for all homogeneous trains in a cycle always equals the cycle time. On the contrary, when traffic on a certain track is highly heterogeneous, then the short distance trains tend to depart just after the long distance trains at the start of the track section, and the long distance trains arrive just after the short distance trains at the end of the track section. This leads to a small total sum of smallest headways. These differences are made explicit and are quantified in the remainder of this section.

The disadvantage of just taking the sum of the smallest headways in a linear way is that it does not take into account how the trains are spread over the cycle time. With a cycle time of 60 minutes with four homogeneous trains, one will always have a total sum of (smallest) headways of 60', whether these trains are nicely spread (four 15'-intervals; Fig. 2(a)) or not (e.g., 5', 25', 5' and 25'-intervals; Fig. 2(b)). However, taking the sum of the reciprocals gives a clear distinction between these situations. In particular, the examples in Fig. 2 lead to  $1/15 + 1/15 + 1/15 + 1/15 = 0.27$ , and  $1/5 + 1/25 + 1/5 + 1/25 = 0.48$ , respectively.

This leads to our first heterogeneity measure, based on both the heterogeneity and the spread of the trains over the hour. This measure is applicable to railway tracks between two neighboring railway nodes. The sum of shortest headway reciprocals (SSHR) is defined as follows:

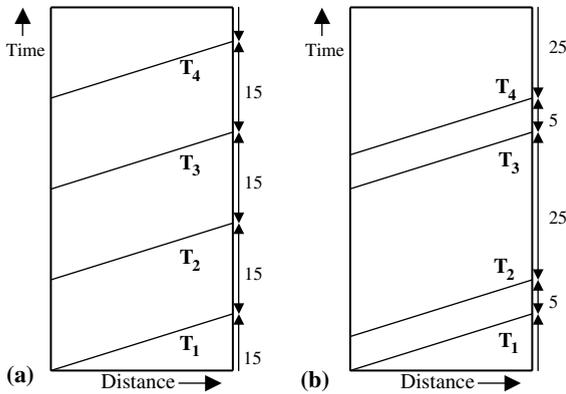


Fig. 2. Time–distance diagram for two homogeneous situations with a different headway distribution.

$$SSHR = \sum_{i=1}^n \frac{1}{h_i^-}, \tag{1}$$

with  $h_i^-$  the smallest scheduled headway between trains  $i$  and  $i + 1$  on the track section, and train  $n$  is followed by train 1, due to cyclicity. It is not difficult to see that, given the number of trains on a certain track section together with their running times, for minimizing the SSHR one should minimize the running time differences between subsequent trains and one should equalize the minimum headways between subsequent trains. If the order of the trains has been prespecified already, then one should equalize the minimum headways between subsequent trains.

As stated earlier, the SSHR is not only capable of representing the distribution of the trains over the hour on a track, but it is also capable of including the heterogeneity of these trains on this track. The homogeneous situation in Fig. 2(a) gives an SSHR of 0.27. The slightly heterogeneous situation in Fig. 3(a) leads to an SSHR of  $1/9 + 1/9 + 1/9 + 1/9 = 0.44$ . Fig. 3(b) represents a very heterogeneous situation with an SSHR of  $1/2 + 1/2 + 1/2 = 2$ .

A disadvantage of the SSHR is that headways at departure are penalized as heavily as headways at arrival. However, headways at arrival seem to be more important than headways at departure. The first reason is that delays at arrival are, on average, larger than delays at departure. Secondly, faster long distance trains can be caught behind

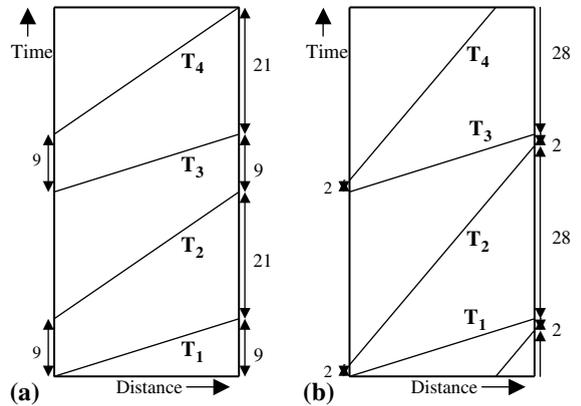


Fig. 3. Two time–distance diagrams with slightly heterogeneous (a) and very heterogeneous (b) railway traffic.

short distance trains towards the end of a railway section. Therefore we propose a second measure, which only depends on the arrival headways between every pair of subsequent trains, the sum of arrival headway reciprocals (SAHR):

$$SAHR = \sum_{i=1}^n \frac{1}{h_i^A}, \tag{2}$$

with  $h_i^A$  the headway at arrival between trains  $i$  and  $i + 1$  on the track section. Again train  $n$  is followed by train 1.

In homogeneous cases, the SAHR is equal to the SSHR, so the SAHR is 0.27 in example 2(a) and 0.48 in example 2(b). In heterogeneous cases, the SAHR is always less than the SSHR. The timetables represented in Fig. 3(a) and (b) have an SAHR of  $1/9 + 1/21 + 1/9 + 1/21 = 0.32$  and  $1/2 + 1/28 + 1/2 + 1/28 = 1.07$ , respectively.

Unfortunately, the SAHR does not take into account the track section anymore and it is in fact a single location measure. Still, the arrival distribution can only be nice if the timetable is not too heterogeneous. This means that heterogeneity is implicitly taken into account in the SAHR. However, an improved measure may be attained by taking the weighted average of the two measures above.

The two measures developed above are not absolute measures. They are mainly meant to be able to compare different timetables for the same track or as an indication how to produce a reliable timetable for a certain track.

### 3.4. Experiments

In this paper we study the effect of homogenization on the reliability and the punctuality of a railway system. For obtaining more homogeneous timetables, we have chosen the option of shifting stops from the short distance services to the long distance services until the number of stops of the trains per track section are as near to equal as possible. This leads to more equal running times per track section.

Cyclic timetables were developed with the automatic timetabling tool DONS (Hooghiemstra, 1994). Any real-life or artificial railway network can be defined with this tool. On this infrastructure, train lines can be defined, including their intermediate stops, their rolling stock types, transfer connections and other characteristics. DONS will then provide a feasible timetable, or, if this is not possible, tell which constraints following from the input make a timetable infeasible.

For the comparison of timetables, simulation of railway traffic has also been used. The simulations reported on in this paper are performed with SIMONE. This simulation model is in use both by ProRail, the Dutch railway infrastructure manager, and by Netherlands Railways, the predominant Dutch operator of passenger trains. It is used both for timetable comparisons and for scientific research. A more detailed description of SIMONE is given by Middelkoop and Bouwman (2000, 2001).

## 4. A theoretical case

The first case that we looked at is a theoretical one. Both the simple network and the involved timetables are artificial. They were developed especially for the comparison of homogeneity and heterogeneity together with the corresponding reliability.

### 4.1. Case description

The network consists of two intersecting double-track lines of 192 km each, which intersect at a Central Station (CS). This creates four identical

branches of 96 km: northwest (NW), northeast (NE), southwest (SW), and southeast (SE). The layout of the network is shown in Fig. 4(a), where the lines do not represent tracks but train lines. The four branches are equal and have three intermediate large stations, where all trains stop. These stations are represented by the rectangles in Fig. 4(b) and (c). Each branch also comprises 10 smaller stations, where half of the trains stop. These smaller stations are closer to each other around CS and around the endpoints. One can consider these areas as more densely populated. The distances were chosen such as to resemble the average station distances in the Netherlands. Fig. 4 shows the stations where a train line stops: Fig. 4(b) represents the heterogeneous situation, and Fig. 4(c) represents the homogeneous situation. Note that the number of trains and the number of stops per station are equal for these two situations.

The intersecting Central Station has free-level crossings only. This means that only trains going to or coming from the same direction can interfere with each other. Still, delays can be transferred throughout the network because of the long distance trains, which alternate in destination. Fig. 4(a) shows which direct connections exist in the experiments: each line represents two trains per direction per hour. There are four *short distance* trains on each branch from CS to the endpoint. Additionally there are four *long distance* trains per hour on each branch, but they alternate in destination: there are two trains per hour from NW96 to SW96, two from NW96 to SE96, two from NE96 to SE96, and two from NE96 to SW96.

Although different types of rolling stock are deployed for short distance trains and for long distance trains in real life, for simplicity only a single type of rolling stock is used in this theoretical case.

### 4.2. Heterogeneous situation

In the heterogeneous situation, the short distance trains stop at all stations and the long distance trains only dwell at the large stations. This dwelling pattern is shown in Fig. 4(b). All trains are nicely spread over the hour, which means that from every station in the system, a short distance

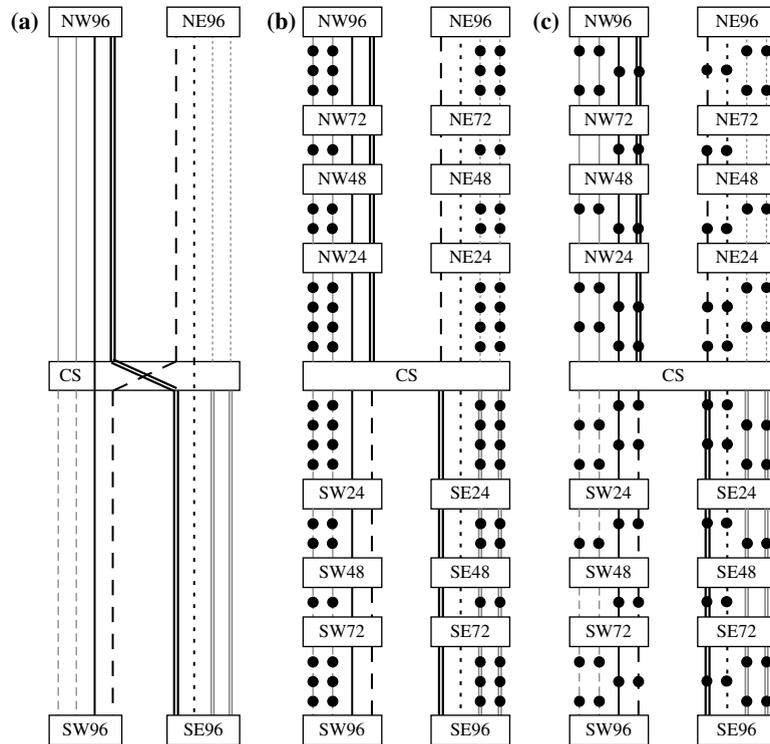


Fig. 4. (a) The different train connections in the network, with the heterogeneous dwelling pattern in (b) and the homogeneous dwelling pattern in (c).

train leaves every 15 minutes in both directions. At the large stations one can also catch a long distance train exactly every 15 minutes, where the individual long distance lines (for example NW96–SW96), run exactly every 30 minutes. A time–distance diagram created by DONS is given in Fig. 5.

The SSHR between CS and one of the endpoints is 5.33 for the heterogeneous case. The SAHR can only be defined for single locations, leading to multiple values. At CS the SAHR is 2.31 from all directions. Coming from CS, the SAHR is also 2.31 at NW48, NE48, SW48 and SE48. Coming from the endpoints, the SAHR is 1.67 at NW48, NE48, SW48 and SE48. Finally, the SAHR is 1.36 at the endpoints.

It should be noted that in heterogeneous situations, overtaking may have to take place, depending on the frequency, the difference in numbers of stops and the time loss per additional stop. In the presented case, overtaking is necessary

indeed: the short distance trains are overtaken by the long distance trains in NW48, NE48, SW48 and SE48. Furthermore, the dwell time of the long distance trains is extended at NW24, NE24, SW24 and SE24 to decrease the travel time differences. Otherwise no feasible timetable would exist for these trains, these dwellings, and this train order.

#### 4.3. Homogeneous situation

In the homogeneous situation, the same number of lines, the same number of stops per station, and the same line–endpoint connections are applied as in the heterogeneous situation. Also the same type of rolling stock is used. However, a more homogeneous situation is created by decreasing the number of stops of the short distance services and by increasing the number of stops of the long distance services until both are as equal as possible. The newly created services are shown in Fig. 4(c).

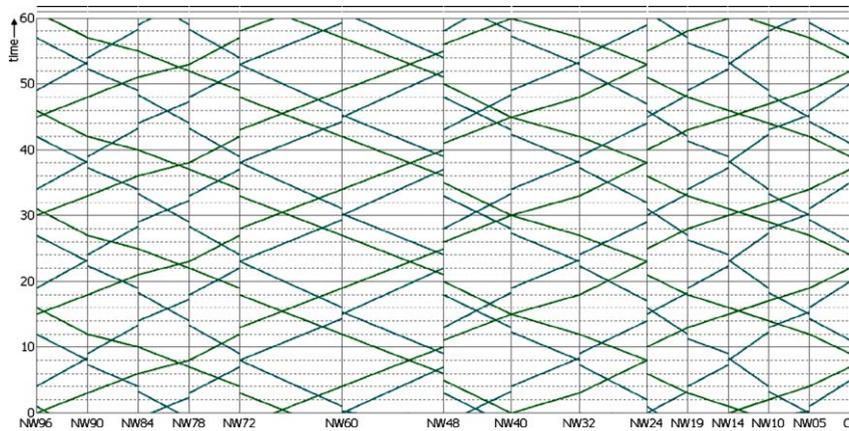


Fig. 5. Time–space diagram of the heterogeneous situation for the branch NW96–CS: there is an apparent difference between the long distance trains—the flatter lines—and the short distance trains—the steeper lines. Vertical jumps in the lines depict stops at stations. Due to acceleration, deceleration and roundings, running times on tracks of equal length may differ from each other. The other three branches have identical time–space diagrams.

Per branch, starting from the endpoints, the long distance lines dwell at the 2nd, 4th, 6th, 8th, and 10th small station, whereas the short distance lines (NW96–CS and so on) dwell at the 1st, 3rd, 5th, 7th, and 9th small station. Again, every small station is served four times an hour, exactly every 15 minutes, and each intercity station is served eight times per hour as is shown in Fig. 6.

The homogeneous situation reduces the heterogeneity measures significantly. For each of the branches, the SSHR goes down from 5.33 to 1.24. At CS the SAHR decreases from 2.31 to 1.11. At

NW48, NE48, SW48 and SE48, we see the SAHR dropping from 2.31 to 1.07 (from CS), and from 1.67 to 1.11 (from the endpoints). The SAHR at the endpoints decreases from 1.36 to 1.07.

#### 4.4. Experimental design

Sixteen experiments with different primary delay distributions were carried out. The primary delays are generated randomly by SIMONE, following the specifications given in Table 1. Dwell times and running times are disturbed with a

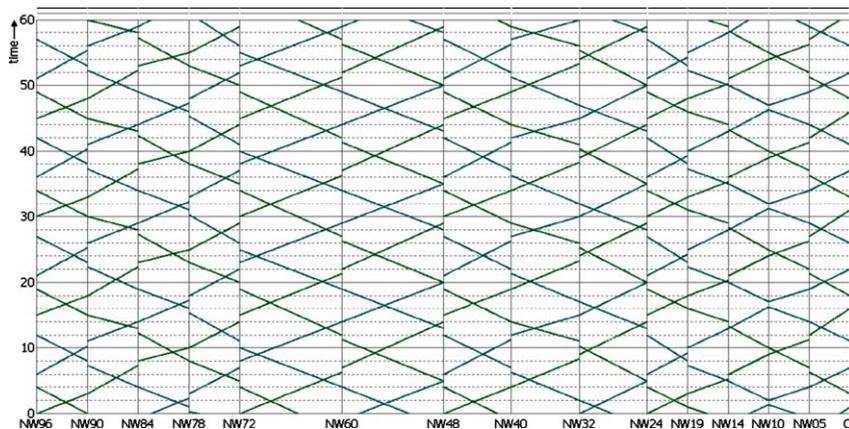


Fig. 6. Time–space diagram of the homogeneous situation: the different trains have almost parallel time–space lines, which leads to a larger minimal headway between trains.

Table 1  
The experiments and their primary disturbances

Experiment	Dwell time disturbance [all stations]		Dwell time disturbance [large stations]		Running time disturbance		Total primary disturbances (in minutes per hour)
	Probability for disturbance (%)	Average disturbance (in minutes)	Probability for disturbance (%)	Average disturbance (in minutes)	Probability for disturbance (%)	Average disturbance (in minutes)	
1	5	1					30.8
2	5	1	5	2			61.7
3	5	1	10	2			92.3
4	5	1	15	2			123.0
5	5	1	20	2			152.9
6	5	1	25	0.5			69.5
7	5	1	25	1			107.6
8	5	1	25	1.5			146.2
9	5	1	25	2			183.4
10			25	1			76.3
11			25	1	20	0.075	128.1
12			25	1	40	0.075	179.5
13			25	1	60	0.075	232.2
14			25	1	10	0.15	128.5
15			25	1	20	0.15	179.3
16			25	1	30	0.15	232.1

certain probability. These disturbances are exponentially distributed with a given average. Earlier research (Goverde et al., 2001) shows that real dwell time disturbances and late arrivals fit well to an exponential distribution.

All simulation experiments consist of 50 runs of 1320 minutes of train operations, including 120 minutes of warm-up time. This leaves exactly 50 times 20 hours of simulated time for which statistics are collected. Twenty hours is close to one day of train services in the Netherlands, where night services are almost non-existing. The use of fifty runs leads to a satisfactory reliability of the simulation results. The obtained average delays have standard deviations up to 3% for the heterogeneous situation and up to 7% for the homogeneous situation over the 50 runs (except for experiment 1, where small probabilities for disturbances cause a higher variability of the results).

The first experiments have a combination of dwell time disturbances at all stations and dwell time disturbances at the large stations. This leads to two cumulative disturbances for the large stations. Experiments 1–5 have an increasing probability for disturbances at the large stations; experiments 1 and 6–9 have an increasing average

size of the dwell time disturbances at the large stations.

The following experiments have a combination of dwell time disturbances at the large stations and running time disturbances. Experiments 11–13 have the same total number of primary delay minutes as experiments 14–16, but the latter experiments have fewer, though larger, disturbances.

#### 4.5. Results

- Table 2 shows that, going from the heterogeneous to the homogeneous situation, the average delays decrease significantly. Although a decrease was expected, the extent is surprisingly large: over 65% in all experiments. Even more surprisingly is the consistency of these results. Although the decrease in unpunctuality varies quite a bit, the average delays always decreased with 66–77 percent.

- The last three columns of Table 2 show where the differences originate from, namely the reduction in the secondary delays. Especially in experiments with only few and small primary delays, there are hardly any secondary delays in the homogeneous situation. A simple explanation for this is the fact that secondary delays only emerge if there

Table 2  
The results of the experiments

Experiment	Average arrival delay (per train measurement in minutes)			3-Minute unpunctuality (% of trains delayed)			Total incurred secondary delays (in minutes)		
	Heterogeneous	Homogeneous	Improvement (relative)	Heterogeneous	Homogeneous	Improvement (relative)	Heterogeneous	Homogeneous	Improvement (relative)
1	0.38	0.09	76.5	2.1	0.5	75.0	36.9	0.5	98.7
2	0.97	0.27	72.4	6.9	2.5	64.2	88.2	4.4	95.0
3	1.57	0.46	70.7	11.7	4.8	59.3	136.0	9.1	93.3
4	2.16	0.65	69.9	16.1	6.9	57.0	177.5	13.4	92.4
5	2.70	0.88	67.5	20.1	9.7	51.7	212.5	18.3	91.4
6	0.64	0.15	76.8	3.8	0.7	82.5	69.9	0.6	99.2
7	1.38	0.33	76.3	9.6	2.3	76.0	129.0	2.1	98.4
8	2.38	0.65	72.5	17.3	6.6	61.8	195.8	9.8	95.0
9	3.21	1.08	66.4	23.6	12.2	48.3	243.1	22.5	90.7
10	0.92	0.21	77.0	5.9	1.2	79.1	83.9	1.1	98.7
11	1.28	0.32	75.3	8.3	1.9	77.0	117.3	1.5	98.8
12	1.73	0.46	73.4	11.4	3.0	73.8	152.6	2.0	98.7
13	2.28	0.66	70.9	15.4	4.7	69.8	188.8	3.0	98.4
14	1.53	0.38	75.0	10.4	2.5	75.8	135.9	2.1	98.5
15	2.20	0.60	72.8	15.4	4.7	69.8	184.4	3.7	98.0
16	2.99	0.88	70.6	21.3	8.0	62.4	234.9	6.0	97.4

The presented average delays have standard deviations between 0.26% and 2.65% in the heterogeneous case, and between 0.76% and 7.00% in the homogeneous case, with the exception of experiment 1 (6.21% and 18.38% respectively).

are “large” primary delays. The smallest planned headway between trains in the homogeneous situation is 6 minutes, and the minimally required headway is only 2 minutes. This allows for at least 4 minutes of primary delay for a certain train before it causes secondary delays to subsequent trains. In the heterogeneous situation, the slightest primary delay causes secondary delays already, because trains are scheduled at minimum headway time after each other.

- When average delays are smaller, the relative improvement is higher. This is probably because the homogeneous situation is relatively immune for small delays, as was explained in the previous paragraph. When delays increase, secondary delays may also occur in the homogeneous situation, thereby reducing the relative difference with the heterogeneous situation.

- Additionally, a few large primary delays are more hurtful to the punctuality than many small primary delays with the same total number of minutes of initial delays. Compare for example experiments 11, 12 and 13 with experiments 14, 15 and 16. The total primary delays in minutes are equal, but the average size of a primary running time delay is twice as large in the latter experiments. This leads to an average delay increase of 20–30%.

Two explanations may be viable here. First, the explanation above is valid again: where a small primary delay is too small to disrupt a second train, this is valid for two small delays on two different trains as well. However, one large delay may have a negative effect on several other trains.

Secondly, two separate primary delays are recovered by two separate running time supplements. Suppose in situation 1, that one train, running from A via B, C and D to E, is delayed 2 minutes at station A and between every pair of stations half a minute can be recovered. Then there will be a 1.5 minute delay at station B, one minute at C, half a minute at D, and the train will arrive on time at station E. The average arrival delay is  $(1.5 + 1 + 0.5 + 0)/4 = 0.75$  minutes. A second train, also running from A to E, is not delayed at all. The average arrival delay of these two trains is 0.375 minutes. Now in situation 2, let us take two trains for the same line, which both have a one-minute primary delay at A. They will both

be 30 seconds delayed at station B, but they will arrive on time at stations C, D and E. This results in an average arrival delay of only  $2 \times (0.5 + 0 + 0 + 0)/8 = 0.125$  minutes.

- The *dwell time disturbance at all stations* has the largest impact on trains with many stops. This means that the expected travel time increase is largest for the short distance trains in the heterogeneous situation. Therefore, the expected travel time differences are even larger than the planned travel time differences.

#### 4.6. Causes for delays in the heterogeneous situation

Finding out where the delays come from may provide useful information. Therefore the average delays over the course of the train services were examined. Fig. 7 shows the average delays on one of the four branches, separated by trains running towards CS (indicated by *a*), and running from CS (indicated by *b*). This is the graph for experiment 3, but it is typical for all experiments in the heterogeneous case.

- The first thing to notice is the fact that the short distance trains are hardly delayed at all, while long distance trains may have considerable delays. This is because a long distance train can

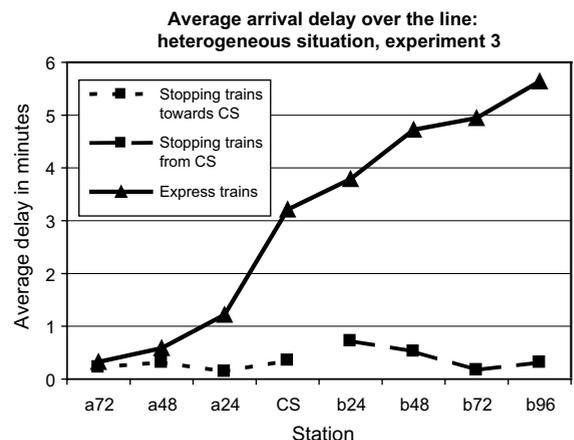


Fig. 7. Average delay over the course of the lines. For example, a72 shows the average arrival delay at NW72, NE72, SW72 and SE72 for trains towards CS. b48 gives the average delay for arrivals at NW48, NE48, SW48 and SE48 for trains going towards the endpoints.

be caught behind a short distance train. In that case, the long distance train can incur large secondary delays, because it cannot run any faster than the short distance train up to the next station where overtaking is possible. However, a short distance train can never be caught behind a slower train. This is exactly why the SAHR was developed next to the SSHR.

The largest delays of short distance trains are found at the first station after starting from CS. This is explained by the delays of the long distance trains in CS, which hamper the departure of the short distance trains starting from CS.

- The average delays of the long distance trains increase with the distance traveled, but not linearly. The largest delay increases can be found just before CS and b48. This is exactly where the heterogeneity causes the headways to be smallest (see Fig. 5) and the long distance trains catch up with the short distance trains. The two other locations where the long distance trains catch up with the short distance trains, a48 and b96, seem to cause less problems. This is due to the lower heterogeneity, and consequently the larger headways, in the peripheral areas.

## 5. A practical case

Besides the theoretical case presented in Section 4, a practical case has been worked out also. We compared a real-life heterogeneous timetable with a more homogeneous timetable for a busy line in the Netherlands. Some details of the real timetable have been adjusted slightly for the simulations (Nederlandse Spoorwegen, 2002).

The case that is described here consists of the lines from The Hague Central (Gvc) and Rotterdam Central (Rtd) to Utrecht Central (Ut), which merge at Moordrecht Junction (Mda) near Gouda (Gd). These lines are represented by the bold lines in Fig. 8. This part of the network has double track everywhere, except for the section between Moordrecht Junction and Gouda Goverwelle (Gdg), which has four parallel tracks. Moordrecht Junction is a non-level crossing. The distances from The Hague to Utrecht and from Rotterdam to Utrecht are 61 km and 55.8 km, respectively.

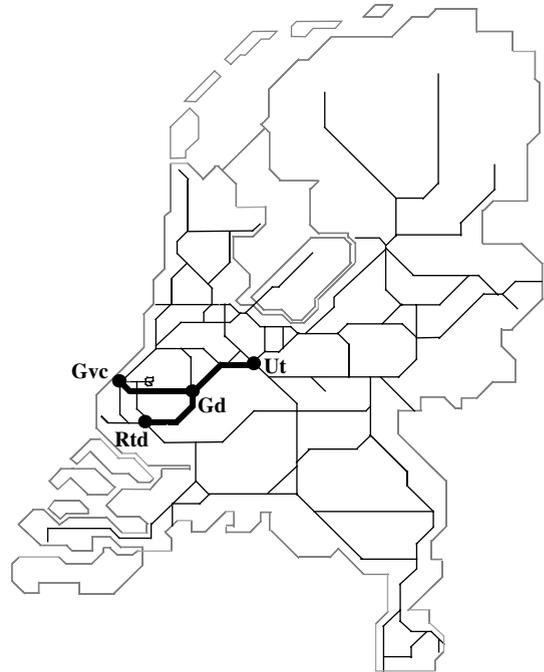


Fig. 8. The railway network served by Netherlands Railways. The bold lines represent the tracks between The Hague and Utrecht and between Rotterdam and Utrecht that are considered in the case.

The lines that are operated on these tracks are shown in Fig. 9. All lines have a cycle time of 30 minutes, which leads to, for example, eight trains per hour between Rotterdam Central and Moordrecht Junction, and 12 trains per hour between Woerden (Wd) and Utrecht Central.

### 5.1. Heterogeneous situation

With some adjustments, the 2003 rush hour timetable has been taken for the heterogeneous situation (Nederlandse Spoorwegen, 2002). Cargo trains are skipped, resulting in a *three-train-system*: long distance trains, interregional trains and short distance trains, represented in Fig. 9 by solid (—), dashed (---), and dotted (···), respectively.

Every 30 minutes there is one short distance train from Gvc to Gdg, there is one interregional train from Gvc to Ut, and there is one long distance train from Gvc to Ut. Starting from Rtd,

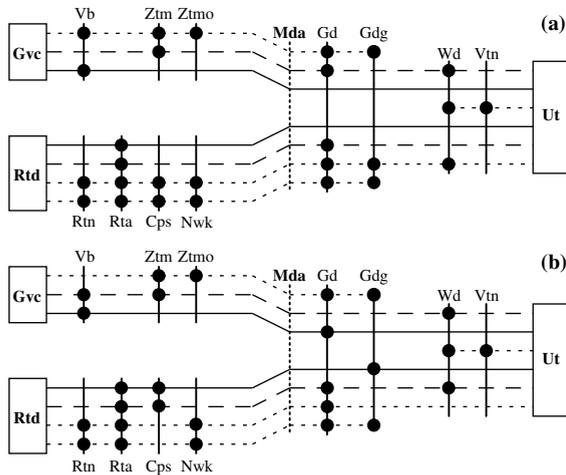


Fig. 9. Train lines and dwelling patterns of the heterogeneous situation (a) and the homogeneous situation (b) of the practical case. Besides the junction Mda, all abbreviations indicate a station.

there is one short distance train running to Gdg, a second short distance train running to Ut (not dwelling at Vleuten (Vtn)), one interregional train to Ut, and one long distance train to Ut. Additionally there is a short distance train from Wd to Ut. This adds up to 16 trains per hour per direction.

Unlike the theoretical case, the lines in this case are run by different rolling stock types. These rolling stock types have their own specific characteristics concerning acceleration and top speed. These are, according to the real-life situation, matched with the service provided.

5.2. Homogeneous situation

As in the theoretical case, the heterogeneous situation is homogenized by decreasing the number

of stops of the short distance services and by compensating those by additional stops of the faster services. In the end, the total number of stops per station is equal in both situations. The final dwell pattern is shown in Fig. 9(b).

The new timetable for the homogeneous situation was generated by the automatic timetabling tool DONS (Hooghiemstra, 1994).

Because of the homogenization, there is no clear distinction anymore between slower and faster services. Therefore, the necessity for different types of rolling stock has gone. However, for a fair comparison, the same rolling stock has been used for both situations. The SSHR and the SAHR for the practical case are given in Table 3.

5.3. Experimental design

Again, the simulation experiments consist of 50 runs of 1320 minutes, including 120 minutes of warm-up time. The primary delay distributions, all exponential again, are given in Table 4.

The first two experiments have dwell time disturbances at all stations. The next four experiments have a combination of dwell time disturbances and absolute running time disturbances. Absolute running time disturbances are independent of the running time and have the averages given in Table 4. The disturbances of experiments 5 and 6 have a larger probability of occurring, but are smaller than those of experiments 3 and 4.

Experiments 7 and 8 have relative running time disturbances, which depend on the scheduled running times of the trains on the track. The average delay equals a certain percentage of the running time. Although the total primary delays are equal,

Table 3  
Tables of SSHR and SAHR for the different tracks between The Hague, Rotterdam and Utrecht

SSHR		Heterogeneous	Homogeneous	SAHR		Heterogeneous	Homogeneous
From	To			From	To		
Gvc	Gd	0.76	0.67	Gvc	Gd	0.64	0.62
Rtd	Gd	3.18	1.47	Rtd	Gd	1.36	1.17
Gd	Ut	3.47	2.97	Gd	Ut	2.70	2.52
Ut	Gdg	5.52	3.02	Ut	Gdg	2.55	1.77
Gd	Gvc	1.06	0.69	Gd	Gvc	0.67	0.61
Gd	Rtd	2.83	1.19	Gd	Rtd	1.74	1.07

Table 4  
The experiments and their primary disturbances

Experiment	Dwell time disturbance [all stations]		Absolute running time disturbance		Relative running time disturbance		Total primary disturbances (in minutes per hour)
	Probability for disturbance (%)	Average disturbance (in minutes)	Probability for disturbance (%)	Average disturbance (in minutes)	Probability for disturbance (%)	Average disturbance (in % of running time)	
1	100	0.6					95.9
2	100	0.8					128.0
3	10	1	5	1			38.7
4	20	1	10	1			78.2
5	50	0.5	20	0.5			86.2
6	75	0.5	30	0.5			129.2
7					30	40	112.8
8					60	20	114.0
9	20	1			30	20	89.3
10	50	0.5			30	20	96.9

experiment 8 has more but smaller disturbances than experiment 7. Experiments 9 and 10 have both dwell time disturbances and relative running time disturbances.

The results of the practical case are shown in Table 5. The results are comparable with those of the theoretical case. The main distinction is that the decrease in delays between the heterogeneous and the homogeneous situation is smaller. This is easily explained by the smaller differences in heterogeneity in the practical case.

#### 5.4. SSHR, SAHR and the results

Furthermore, Fig. 10(a) and (b) show the improvements in the SSHR and in the SAHR in comparison with the delay reductions. For this comparison, the network has been divided into six sections: Gvc–Gd, Rtd–Gd and Gd–Ut and vice versa. The average delay for each section, as given in the figures, is the average delay of all trains at the endpoint of the section minus the average delay of these trains at the start of the section. Thus it is the *increase* in average delay on the corresponding section.

The upper right endpoint of each line-segment in Fig. 10(a), represented by a circle, is the SSHR and the average delay in the homogeneous situation. The other endpoint, represented by a square,

is the result of the homogenization. Fig. 10(b) shows the same comparison for the SAHR. The graphs represent the results of experiment 7, but the graphs for the other experiments are quite similar.

- Fig. 10(a) and (b) show that homogenization of the timetable leads to a reduction both in the SSHR and in the SAHR. It also shows that it leads to a reduction in the average delays on all track sections.
- The decrements in the SSHR are quite different for the different track sections: Gvc–Gd only shows a small difference, while Ut–Gd shows a large decrease. The same is true for the SAHR.
- The relative reductions in the SSHR are not equal, or almost equal, to those of the SAHR. This means that the SSHR and the SAHR are two rather distinct measures. See for example the track section Rtd–Gd.
- In general, a larger decrease in the SSHR leads to a larger reduction in the delays. Still, in Fig. 10(a) the line-segments for the track sections Rtd–Gd and Gd–Ut are rather flat. This means that the delays do not always decrease as much as the SSHR suggests. Therefore the SSHR can be used as an indication in what direction the reliability will move, but it is not an absolute measure.

Table 5  
The results of the practical case

Experiment	Average arrival delay (per train measurement in minutes)		3-Minute unpunctuality (% of trains delayed)		Total incurred secondary delays (in minutes per hour)		Improvement (relative in %)
	Heterogeneous	Homogeneous	Heterogeneous	Homogeneous	Heterogeneous	Homogeneous	
1	1.08	0.72	8.8	3.1	30.0	3.2	89.3
2	1.85	1.22	21.1	10.4	50.3	8.4	83.4
3	0.60	0.40	5.0	3.0	20.9	4.2	80.2
4	1.40	0.92	15.0	9.0	41.3	10.8	74.0
5	1.13	0.75	8.7	3.9	28.8	3.9	86.6
6	2.02	1.35	23.1	12.0	47.5	8.7	81.7
7	2.21	1.55	25.9	17.6	60.2	22.9	62.0
8	1.71	1.22	18.3	10.8	38.7	10.4	73.1
9	1.30	0.89	12.4	7.5	35.0	8.7	75.2
10	1.28	0.88	11.4	5.9	32.1	6.3	80.4

The average delays have standard deviations between 0.3% and 2.4% in the heterogeneous case, and between 0.3% and 3.3% in the homogeneous case.

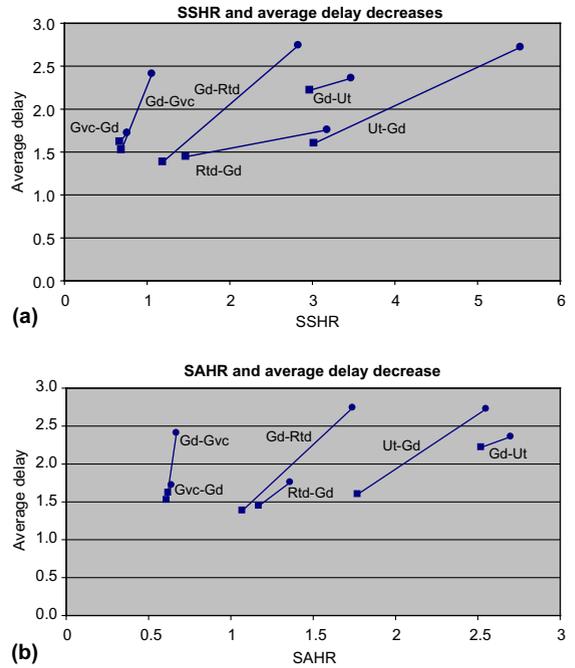


Fig. 10. (a) The relation between the decrease in SSHR and the reduction of the average delay. (b) The relation between the decrease in SAHR and the reduction of the average delay.

For the starting sections, the departure reliability from the first stations is very high. This means that small departure headways hardly have a negative influence on the reliability. The reduction of the SSHR on Rtd–Gd is mainly based on a fairer distribution of the departures; arrivals are hardly affected by the homogenization. This may explain the relatively small reliability improvement of this section.

- Also, a larger decrease in the SAHR leads to a larger reduction in delays. Still, sections with a relatively large delay reduction can be observed (e.g. Gd–Gvc), as well as sections with a relatively small delay reduction (e.g. Gd–Ut).

Due to the large reduction of the delays on the section Ut–Gd, predicted by the SAHR, there is also a large reduction in the departure delays for the section Gd–Gvc. A better starting reliability implies fewer secondary delays, which explains the large delay reduction on Gd–Gvc, and, to a lesser extent, on Gd–Rtd.

## 6. Discussion

### 6.1. The measures: SSHR and SAHR

The measures SSHR and SAHR are both able to predict reliability changes. The SSHR is applicable to track sections between stations, whereas the SAHR can be used for a station, or for all arrivals from a certain track at that station.

Although a large decrease in the SSHR or in the SAHR leads to a large decrease in delays in most cases, it is hard to predict the exact size of the delay reduction. Using a weighted average of the two measures may be advantageous, because it takes heterogeneity into account, and it weighs the arrival headways more heavily than the departure headways.

### 6.2. Equalizing headways

Minimizing the SSHR or the SAHR implies equalizing the headways. Although a reduction in these measures indicates a reduction in delays, minimizing the sum of SSHRs or SAHRs over the network is not necessarily optimal. This can for example be seen from the SSHR in the practical case, where a large reduction on one section (Rtd–Gd) has much less influence on the reliability than the small reduction of the SSHR on another one (Gd–Gvc).

### 6.3. Utilize and build

“Benutten & Bouwen” (“Utilize & Build”) is the vision and the intended direction of the combined Dutch railway branch for the future, up to 2020. Experts from the ministry of Traffic and Waterworks, the railway infrastructure manager ProRail, the passenger operator Netherlands Railways, and the cargo operator Railion (Nederlandse Spoorwegen et al., 2003) have participated in this project. The main problem is how to facilitate the expanding railway traffic on the limited infrastructure. The starting point for the project is to better utilize the existing infrastructure, which is made feasible by means of small but smart infrastructure investments.

Homogenization of the railway system is one of the basic elements of “Benutten & Bouwen”. Although the main focus is on a limited homogenization where intercity trains stop at a few more stations of medium size, full homogenization is also discussed and not a priori excluded as a solution.

### 6.4. Other consequences of homogenization

Although reliability will increase when train services are homogenized, there are several other important characteristics to be considered both for passengers and operators. Homogenization can have its influence on many of those characteristics.

- *Travel time* for passengers is an important determinant of *service quality* in case of homogenization. The planned travel time may decrease for some passengers, but increase for others. The *number of passenger transfers* and the *transfer times* may also change. This requires a further mobility analysis, which falls outside the scope of this paper.
- *Infrastructural needs* can possibly change due to other train lengths, but also due to other locations for overtakings, and due to another way of coordinating trains at large transfer stations.
- When the timetable is homogenized, the *rolling stock* can be standardized as well. The total required number of rolling stock units can also change.
- Homogenization by one large operator may lead to *additional time-slots* in the timetable, which might be assigned to other operators. Evidently this would, in the end, lead to an increased SSHR and SAHR. Therefore, network wide cooperation is necessary for a beneficial introduction of a homogenized timetable.
- Besides timetabling, the rolling stock and crew schedules can also have a considerable influence on delay propagation.

## 7. Conclusions

In this paper the sum of shortest headway reciprocals (SSHR) and sum of arrival headway

reciprocals (SAHR) were described. These measures are used for evaluating the heterogeneity of the timetable and for predicting the reliability. The SSHR can be applied to a whole section and has the desirable property that it decreases both when trains are spread better over the hour and when railway traffic is more homogeneous. The SAHR also has the property of decreasing when the trains are spread better over the hour. It lacks a direct link to the heterogeneity, but takes it into account implicitly.

The presented cases show a large reliability increase for homogenized services with low values for the SAHR and SSHR. In other words, when the SSHR and SAHR show large decreases, then there are usually also large decreases in delay propagation. Therefore, a relatively simple rule of thumb for timetable design is to minimize the SSHR and the SAHR. Our experiments show that this is likely to improve the reliability of the offered services.

Although homogenization may lead to a sizable increase in punctuality of the offered railway services, homogenization may also effect other features of the railway product, both for passengers and for operators and infrastructure managers. When homogenizing train services, these other consequences should also be considered. This is a subject for further research. The relationship between the consequences for the different operators and the infrastructure managers also stresses the importance of cooperation between these parties.

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